FACULTY
OF MATHEMATICS
AND PHYSICS
Charles University

Finite-coupling spectrum of O(N) model in AdS

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MINISTRY OF EDUCATION, YOUTH AND SPORTS

FORTE

O(N) model

N scalar fields $\{\phi^i\}$

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$$\mathcal{S}[\phi^\bullet] = \int_{\mathcal{M}} \mathrm{d}^{d+1}x \sqrt{g} \left[\frac{1}{2}(\partial\phi^\bullet)^2 + \frac{1}{2}m^2(\phi^\bullet)^2 \right]$$

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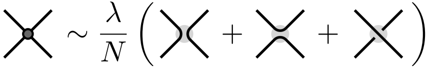

O(N) model

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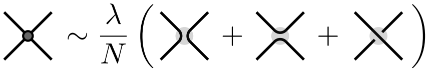

large N expansion

O(N) model

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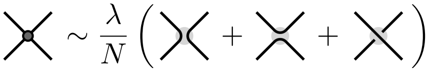

large N expansion $\overset{\text{allows}}{\rightsquigarrow}$ finite coupling λ

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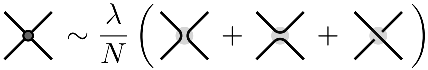

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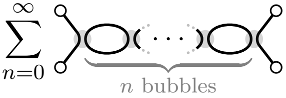

O(N) model

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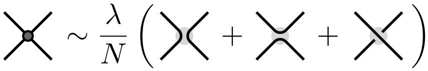

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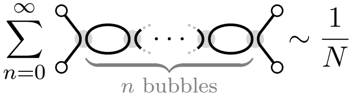

O(N) model in AdS

fixed AdS background

O(N) model in AdS

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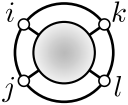

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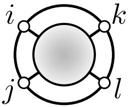
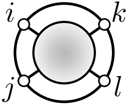


$$= \left\langle \mathcal{O}^i \mathcal{O}^j \mathcal{O}^k \mathcal{O}^l \right\rangle$$

O(N) model in AdS

fixed AdS background

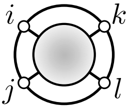


$$= \left\langle \mathcal{O}^i \mathcal{O}^j \mathcal{O}^k \mathcal{O}^l \right\rangle$$

$$\mathcal{O}^i(P) \sim \lim_{s\to\infty} s^{\Delta_\phi} \phi^i\Big(X \equiv sP + \ldots\Big)$$

O(N) model in AdS

fixed AdS background



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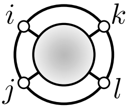
CFT

$\mathsf{O}(N)$ model in AdS

fixed AdS background



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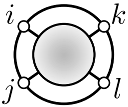
QFT in AdS $\longleftrightarrow$ CFT

$\mathbf{O}(N)$ model in AdS

fixed AdS background



$$= \left\langle \mathcal{O}^i \mathcal{O}^j \mathcal{O}^k \mathcal{O}^l \right\rangle$$

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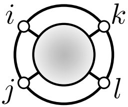
QFT in AdS $\longleftrightarrow$ CFT

CFT in AdS_{d+1}

O(N) model in AdS

fixed AdS background



$$= \left\langle \mathcal{O}^i \mathcal{O}^j \mathcal{O}^k \mathcal{O}^l \right\rangle$$

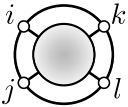
$$\mathcal{O}^i(P) \sim \lim_{s\to\infty} s^{\Delta_\phi} \phi^i\Big(X \equiv sP + \dots\Big)$$

QFT in AdS $\longleftrightarrow$ CFT

CFT in AdS$_{d+1}$ $\xleftrightarrow{\text{Weyl transformation}}$

$\mathsf{O}(N)$ model in AdS

fixed AdS background



$$= \left\langle \mathcal{O}^i \mathcal{O}^j \mathcal{O}^k \mathcal{O}^l \right\rangle$$

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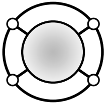
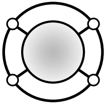
$$\textsf{QFT in AdS} \longleftrightarrow \textsf{CFT}$$

$$\textsf{CFT in AdS}_{d+1} \xleftrightarrow{\text{Weyl transformation}} \textsf{BCFT in } \mathbb{R}^d \times \mathbb{R}_{\geq}$$

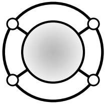
Conformal Block/Partial Wave Decomposition



Conformal Block/Partial Wave Decomposition

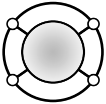
 $\equiv \langle(\mathcal{O}\mathcal{O})(\mathcal{O}\mathcal{O})\rangle$

Conformal Block/Partial Wave Decomposition

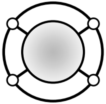


$$\equiv \langle (\mathcal{O}\mathcal{O})(\mathcal{O}\mathcal{O}) \rangle = \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \mathsf{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \left| G^{(s)}_{\Delta_\star, J_\star} \right\rangle$$

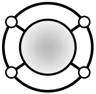
Conformal Block/Partial Wave Decomposition



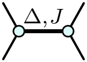
$$\equiv \langle (\mathcal{O}\mathcal{O})(\mathcal{O}\mathcal{O})\rangle = \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \mathsf{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \Big| G^{(s)}_{\Delta_\star,J_\star} \Big\rangle$$



$$= \sum_{J=0}^{\infty} \int_{\frac{d}{2}+i\mathbb{R}_\geq} \frac{\mathrm{d}\Delta}{2\pi i} \, \mathsf{Spec}_s \left[\begin{matrix}\Delta \\ J\end{matrix} \right|$$

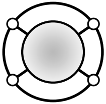


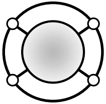
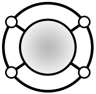
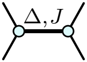
$$\Big] \Big|$$

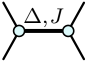


$$\Big\rangle$$

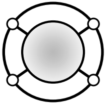
Conformal Block/Partial Wave Decomposition

 $\equiv \langle (\mathcal{O}\mathcal{O})(\mathcal{O}\mathcal{O})\rangle = \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \mathsf{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \Big| G^{(s)}_{\Delta_\star,J_\star} \Big\rangle$

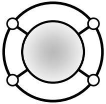
 $= \sum_{J=0}^{\infty} \int_{\frac{d}{2}+i\mathbb{R}_\geq} \frac{\mathrm{d}\Delta}{2\pi i} \, \mathsf{Spec}_s \left[\begin{array}{c} \Delta \\ J \end{array} \right|$  $\Big] \Big|$  $\Big\rangle$

$\Big|$  $\Big\rangle = K_{\widetilde{\Delta},J} \Big| G^{(s)}_{\Delta,J} \Big\rangle \quad + K_{\Delta,J} \Big| G^{(s)}_{\widetilde{\Delta},J} \Big\rangle$

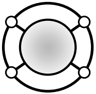
Conformal Block/Partial Wave Decomposition



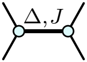
$$\equiv \langle (\mathcal{O}\mathcal{O})(\mathcal{O}\mathcal{O})\rangle = \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \mathsf{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \Big| G^{(s)}_{\Delta_\star, J_\star} \Big\rangle$$



$$= \sum_{J=0}^{\infty} \int_{\frac{d}{2}+i\mathbb{R}_\geq} \frac{\mathrm{d}\Delta}{2\pi i} \, \mathsf{Spec}_s \left[\begin{matrix} \Delta \\ J \end{matrix} \,\middle|\,$$

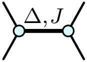


$$\right] \Big|$$



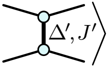
$$\Big\rangle$$

$$\Big|$$



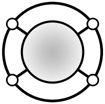
$$\Big\rangle = K_{\widetilde{\Delta}, J} \Big| G^{(s)}_{\Delta, J} \Big\rangle + K_{\Delta, J} \Big| G^{(s)}_{\widetilde{\Delta}, J} \Big\rangle$$

$$\Big|$$

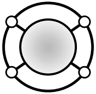


$$\Big\rangle = K_{\widetilde{\Delta}', J'} \Big| G^{(t)}_{\Delta', J'} \Big\rangle + K_{\Delta', J'} \Big| G^{(t)}_{\widetilde{\Delta}', J'} \Big\rangle$$

Conformal Block/Partial Wave Decomposition

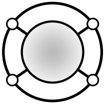


$$= \sum_J \int_{\frac{d}{2}+i\mathbb{R}} \frac{\mathrm{d}\Delta}{2\pi i} \, \mathsf{Spec}_s \left[\begin{matrix} \Delta \\ J \end{matrix} \, \middle| \right.$$



$$\left. \vphantom{\begin{matrix} \Delta \\ J \end{matrix}} \right] K_{\widetilde{\Delta},J} \left| G^{(s)}_{\Delta,J} \right\rangle$$

Conformal Block/Partial Wave Decomposition



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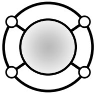
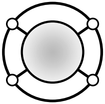


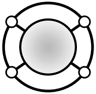
$$\left. \right] K_{\widetilde{\Delta},J} \left| G^{(s)}_{\Delta,J} \right\rangle$$

$$= \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \mathsf{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \left| G^{(s)}_{\Delta_\star, J_\star} \right\rangle$$

Conformal Block/Partial Wave Decomposition



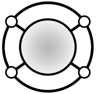
$$= \sum_J \int_{\frac{d}{2}+i\mathbb{R}} \frac{\mathrm{d}\Delta}{2\pi i}\, \mathsf{Spec}_s\left[\begin{matrix}\Delta\\ J\end{matrix}\,\middle|\right.$$



$$\left.\right]\, K_{\widetilde{\Delta},J}\left|G^{(s)}_{\Delta,J}\right\rangle$$

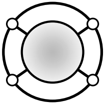
$$= \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \mathsf{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star]\left|G^{(s)}_{\Delta_\star,J_\star}\right\rangle$$

$$\frac{-C_\star}{\Delta-\Delta_\star}\,\delta_{J,J_\star} \in K_{\widetilde{\Delta},J}\, \mathsf{Spec}_s\left[\begin{matrix}\Delta\\ J\end{matrix}\,\middle|\right.$$

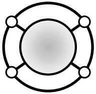


$$\left.\right]$$

Conformal Block/Partial Wave Decomposition



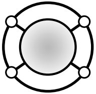
$$= \sum_J \int_{\frac{d}{2}+i\mathbb{R}} \frac{\mathrm{d}\Delta}{2\pi i}\, \mathsf{Spec}_s\left[\begin{matrix}\Delta\\ J\end{matrix}\;\right|$$



$$\Bigg]\; K_{\widetilde{\Delta},J}\left|G^{(s)}_{\Delta,J}\right\rangle$$

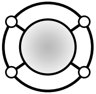
$$= \sum_{\substack{\text{primary } \mathcal{O}_\star \\ \text{with } \Delta_\star, J_\star}} \mathsf{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star]\left|G^{(s)}_{\Delta_\star,J_\star}\right\rangle$$

$$\frac{-C_\star}{\Delta-\Delta_\star}\,\delta_{J,J_\star} \in K_{\widetilde{\Delta},J}\,\mathsf{Spec}_s\left[\begin{matrix}\Delta\\ J\end{matrix}\;\right|$$



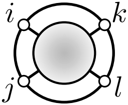
$$\Bigg]$$

$$\mathsf{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \equiv C_\star = -\,\mathrm{Res}_{\Delta=\Delta_\star}\left(K_{\widetilde{\Delta},J_\star}\,\mathsf{Spec}_s\left[\begin{matrix}\Delta\\ J_\star\end{matrix}\;\right|\right.$$

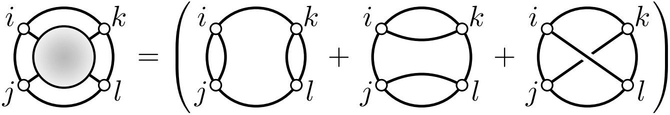


$$\left.\Bigg]\right)$$

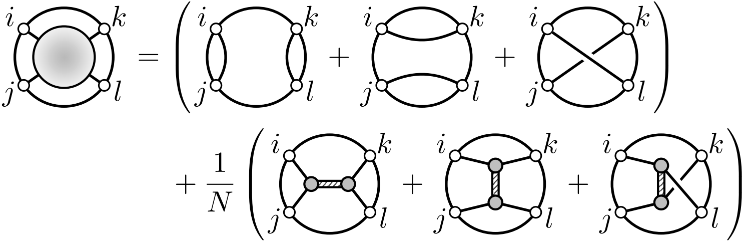
Boundary 4-point correlator in AdS



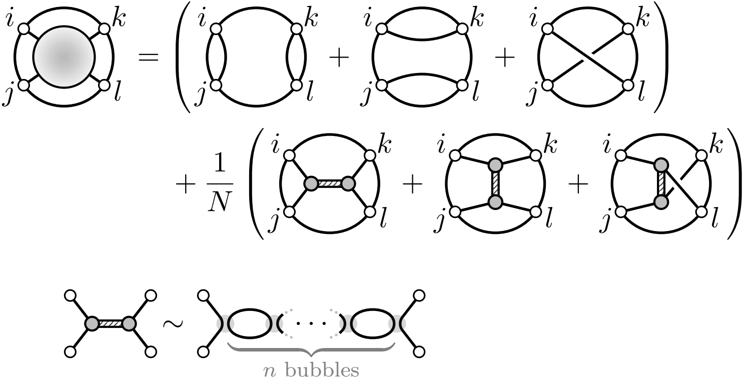
Boundary 4-point correlator in **AdS**



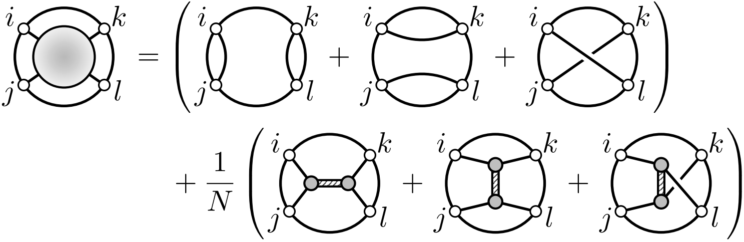
Boundary 4-point correlator in **AdS**

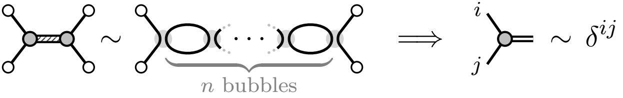


Boundary 4-point correlator in **AdS**

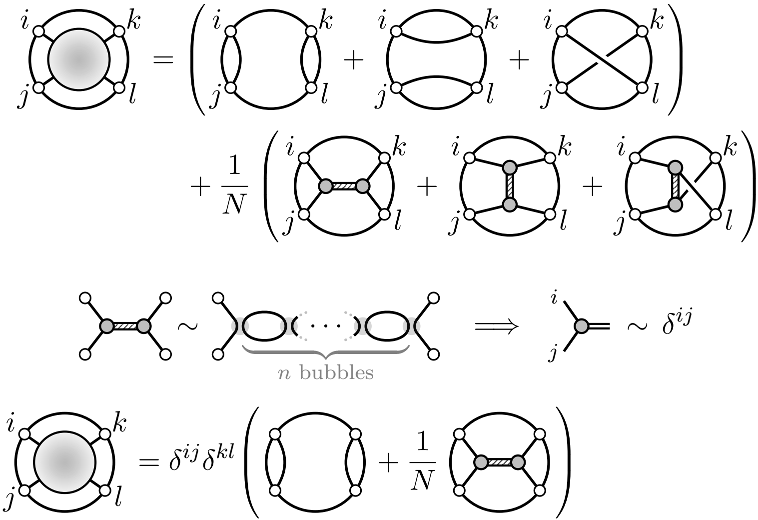


Boundary 4-point correlator in AdS

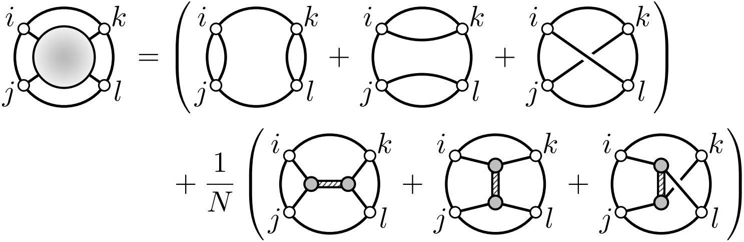


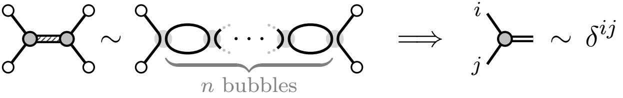


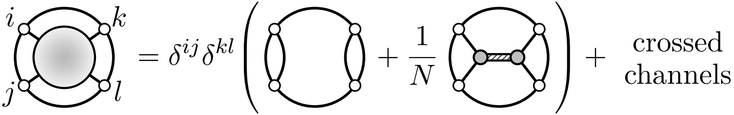
Boundary 4-point correlator in **AdS**



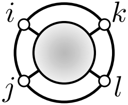
Boundary 4-point correlator in **AdS**



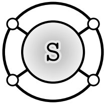




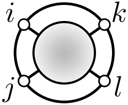
Decomposition into $\mathbf{O}(N)$ irreps



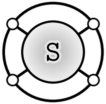
$$= \overbrace{N\mathcal{P}^{ijkl}_{(\mathrm{S})}}^{\delta^{ij}\delta^{kl}}$$



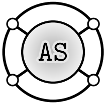
Decomposition into $\mathsf{O}(N)$ irreps



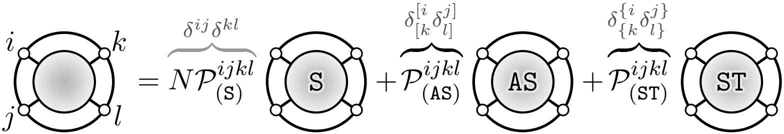
$= \overbrace{N\mathcal{P}^{ijkl}_{(\mathtt{S})}}^{\delta^{ij}\delta^{kl}}$



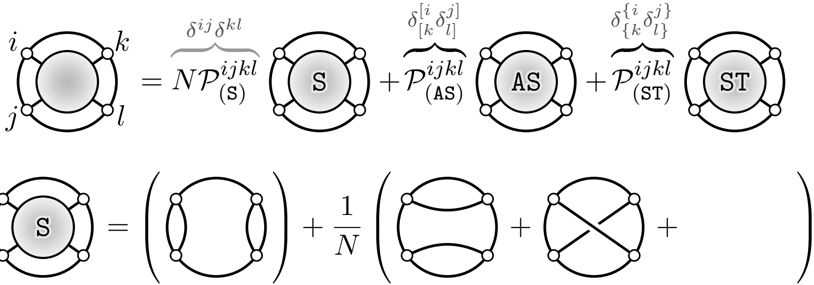
$+ \overbrace{\mathcal{P}^{ijkl}_{(\mathtt{AS})}}^{\delta^{[i}_{[k}\delta^{j]}_{l]}}$



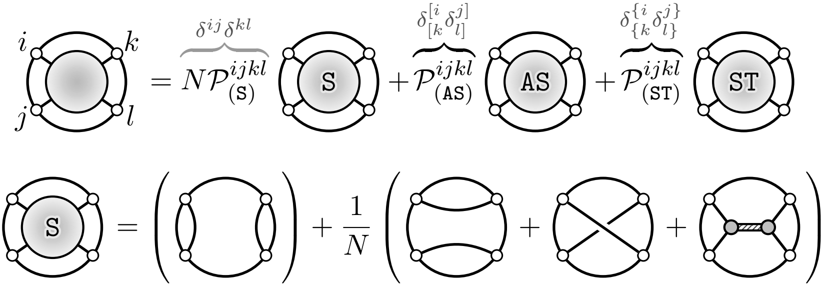
Decomposition into $\mathbf{O}(N)$ irreps



Decomposition into $\mathbf{O}(N)$ irreps



Decomposition into $\mathbf{O}(N)$ irreps



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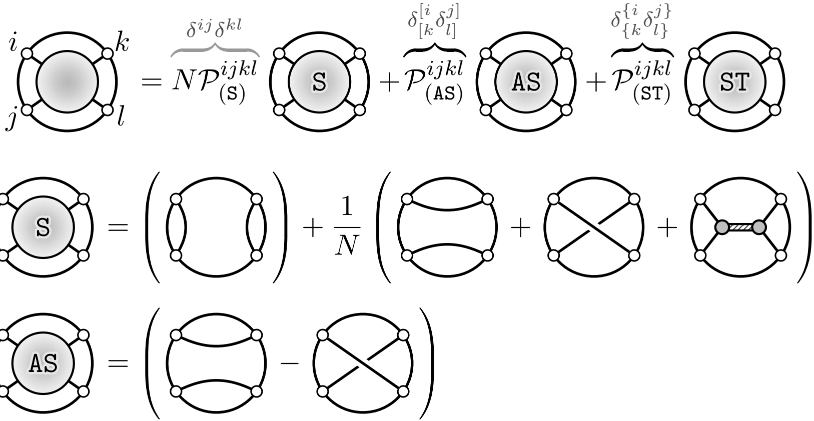

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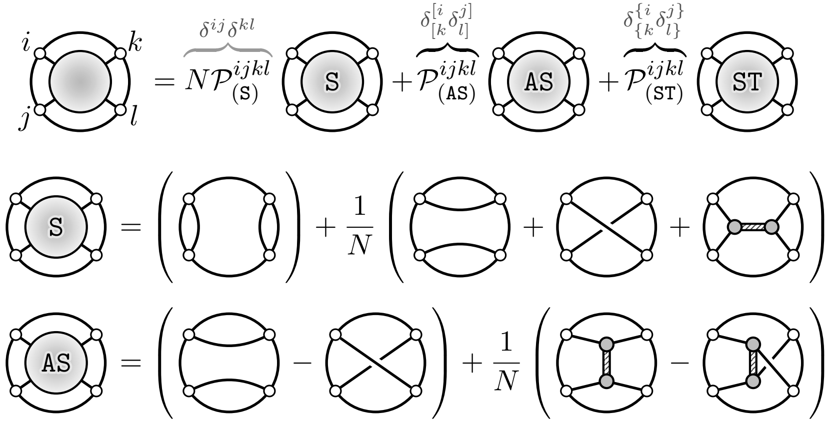

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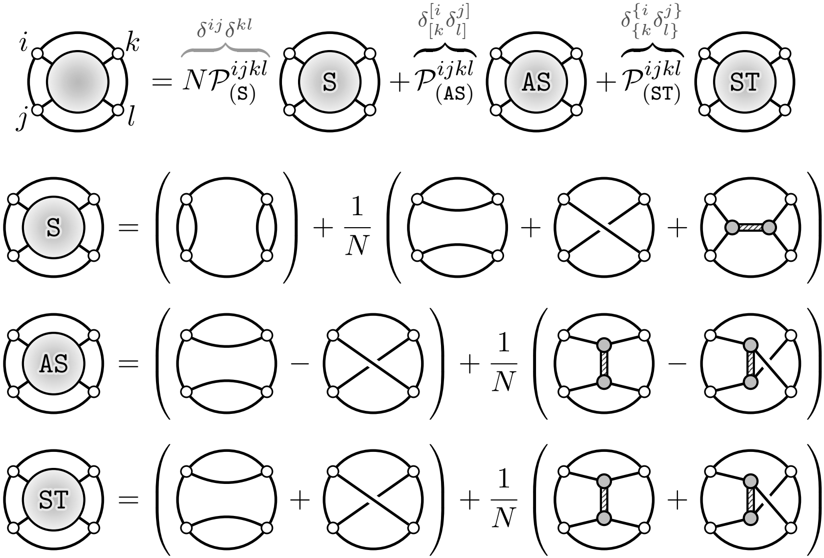
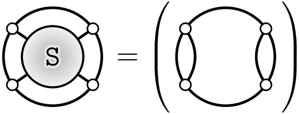


Singlet sector (MFT)

Analyzed in D. Carmi, L. Di Pietro and S. Komatsu, *A Study of Quantum Field Theories in AdS at Finite Coupling*, *JHEP* **01** (2019) 200 [1810.04185]



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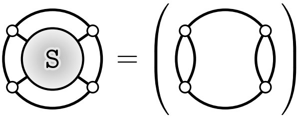

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(\mathrm{S})} \overset{N=\infty}{\sim} \mathbb{1}^{(\mathrm{S})}$$

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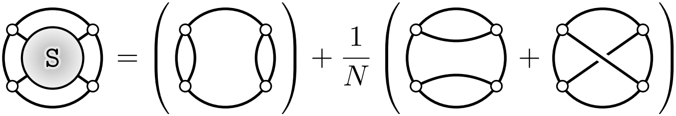

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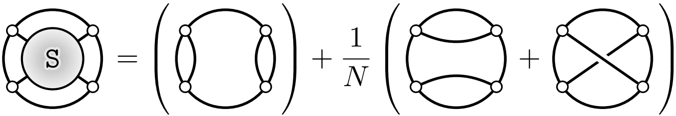

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(\mathtt{S})} \overset{N=\infty}{\sim} \mathbb{1}^{(\mathtt{S})}$$

$$[\mathcal{O}^i \times \mathcal{O}^j]^{(\mathtt{S})} \overset{\lambda=0}{\sim} \mathbb{1}^{(\mathtt{S})} \oplus \frac{1}{\sqrt{N}} \left[\mathcal{O}^\bullet \Box^n \partial^J_{\text{even}} \mathcal{O}^\bullet\right]^{(\mathtt{S})}$$

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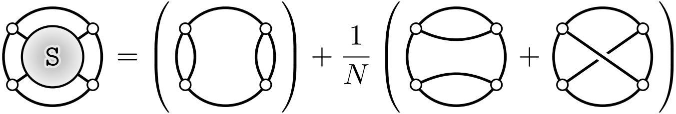

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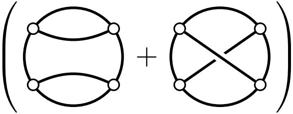

$$= \sum_{n=0}^{\infty} \sum_{\substack{J=0 \\ J \text{ even}}}^{\infty} 2C^{(\text{MFT})}_{n,J} \left| G^{(s)}_{2\Delta_\phi+2n+J,J} \right\rangle$$

Singlet sector (interacting)

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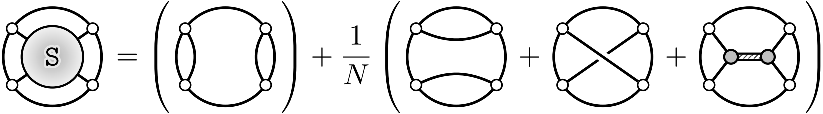

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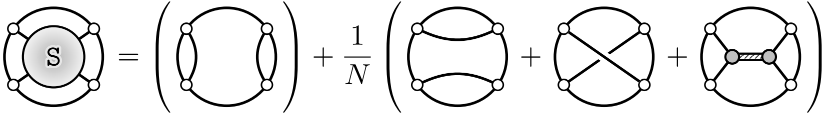

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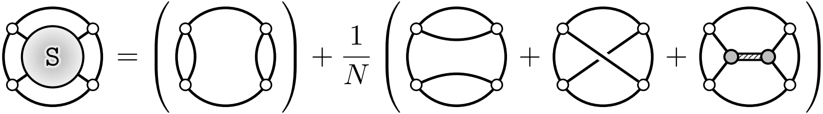

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$$[\mathcal{O}^i \times \mathcal{O}^j]^{(\mathrm{S})} \sim \mathbb{1}^{(\mathrm{S})} \oplus \frac{1}{\sqrt{N}} [\text{“}\mathcal{O}^{\bullet} \Box^n \mathcal{O}^{\bullet}\text{”}]^{(\mathrm{S})}_{O(1)\ \text{finite shifts}}$$

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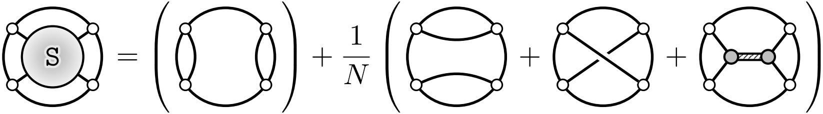

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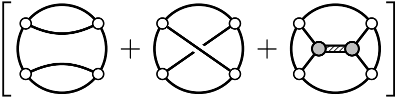
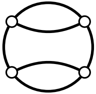
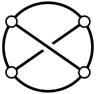
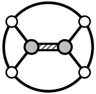
$$\oplus \frac{1}{\sqrt{N}} \left[\mathcal{O}^{\bullet} \Box^n \partial^{J>0}_{\text{even}} \mathcal{O}^{\bullet}\right]^{(\mathrm{S})}_{\text{MFT}}$$

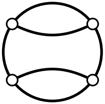
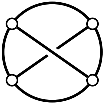
Singlet sector — spectral functions

$$\frac{1}{N}\,\mathsf{Spec}_s$$

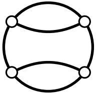
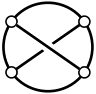
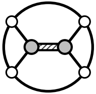


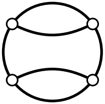
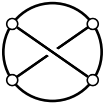
Singlet sector — spectral functions

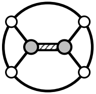
$$\frac{1}{N}\,\mathsf{Spec}_s\Bigg[$$  $$+$$  $$+$$  $$\Bigg]$$

$$\Bigg($$  $$+$$  $$\Bigg) = \sum_{n=0}^{\infty} \sum_{\substack{J=0 \\ J \text{ even}}}^{\infty} 2C_{n,J}^{(\mathrm{MFT})} \Big| G^{(s)}_{2\Delta_\phi+2n+J,J} \Big\rangle$$

Singlet sector — spectral functions

$\frac{1}{N}\,\mathsf{Spec}_s\Big[$  $+$  $+$  $\Big]$

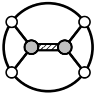
$\Big($  $+$  $\Big) = \sum_{n=0}^{\infty} \sum_{\substack{J=0 \\ J\ \mathrm{even}}}^{\infty} 2C_{n,J}^{(\mathrm{MFT})} \Big| G^{(s)}_{2\Delta_\phi+2n+J,J} \Big\rangle$

$\mathsf{Spec}_s\left[\begin{array}{c} \Delta \\ J \end{array}\middle|\right.$  $\Big] = -\delta_{J,0}\ \frac{1}{\lambda^{-1}+2\widetilde{B}(\Delta)} \times$

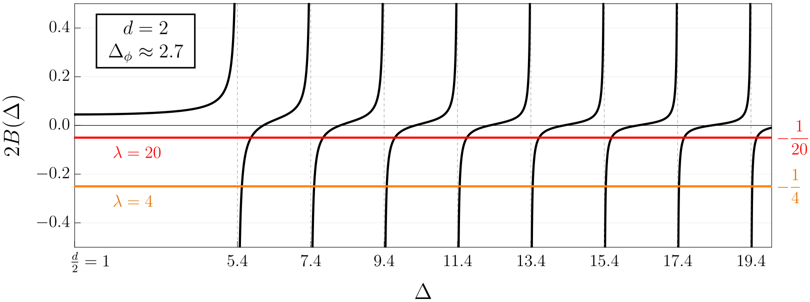
$$\times\ \frac{\Gamma^2_{\Delta_\phi-\frac{\Delta}{2}}\,\Gamma^2_{\Delta_\phi-\frac{\widetilde{\Delta}}{2}}\,\Gamma^2_{\frac{\Delta}{2}}\,\Gamma^2_{\frac{\widetilde{\Delta}}{2}}}{4\pi^d\,\Gamma^2_{\Delta_\phi}\,\Gamma^2_{1-\frac{d}{2}+\Delta_\phi}\,\Gamma_{\Delta-\frac{d}{2}}\,\Gamma_{\widetilde{\Delta}-\frac{d}{2}}}$$

Singlet sector — scalar non-MFT operators

$$\mathsf{Spec}_s\left[\begin{array}{c|c}\Delta & \\ J & \end{array}\right] \propto \frac{1}{\lambda^{-1}+2\widetilde{B}(\Delta)}$$

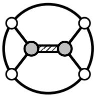


$$\implies \lambda^{-1}+2\widetilde{B}\left(\Delta^{(\mathsf{S})}_{\bullet,0}\right)=0$$



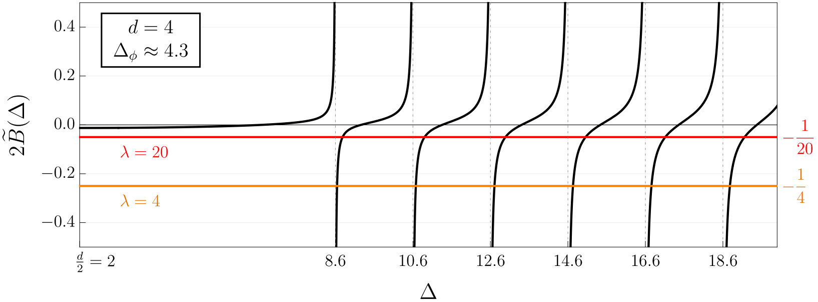
Singlet sector — scalar non-MFT operators

$$\mathsf{Spec}_s \left[\begin{matrix} \Delta \\ J \end{matrix} \middle| \right.$$

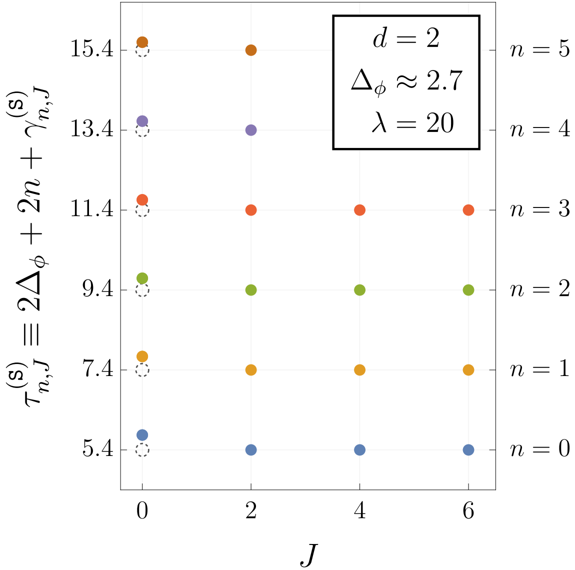


$$\left. \right] \propto \frac{1}{\lambda^{-1} + 2\widetilde{B}(\Delta)}$$

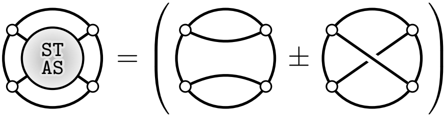
$$\Longrightarrow \quad \lambda^{-1} + 2\widetilde{B}\left(\Delta^{(\mathsf{S})}_{\bullet,0}\right) = 0$$



Singlet sector — twist–spin plot



Non-singlet sector (MFT)



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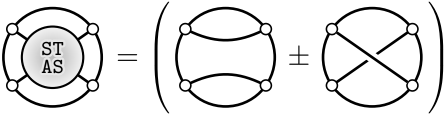
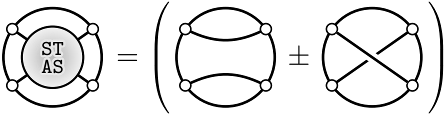


$$\left[\mathcal{O}^i \times \mathcal{O}^j\right]^{(\mathtt{ST})} \overset{N=\infty}{\sim} \left[\mathcal{O}^{\{i} \Box^n \partial^J_{\text{even}} \mathcal{O}^{j\}}\right]^{(\mathtt{ST})}$$

Non-singlet sector (MFT)



$$\left[\mathcal{O}^i \times \mathcal{O}^j\right]^{(\texttt{ST})} \overset{N=\infty}{\sim} \left[\mathcal{O}^{\{i} \Box^n \partial^J_{\text{even}} \mathcal{O}^{j\}}\right]^{(\texttt{ST})}$$

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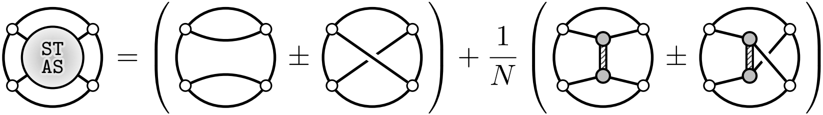
Non-singlet sector (interacting)



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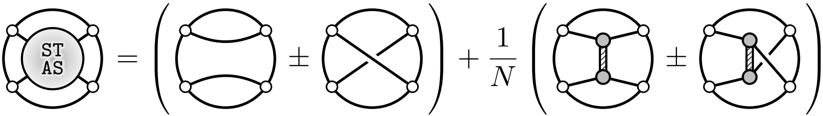

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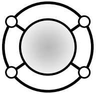
Anomalous dimensions as double poles

$$\frac{-C_\star}{\Delta - \Delta_\star}\,\delta_{J,J_\star} \in K_{\widetilde{\Delta},J}\,\mathsf{Spec}_s\left[\begin{array}{c}\Delta\\ J\end{array}\middle|\; \cdots \;\right]$$



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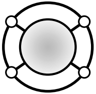
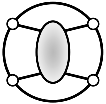
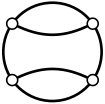


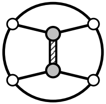
$$\left.\right]$$



$$=$$



$$+\frac{1}{N}$$



$$+\ldots$$

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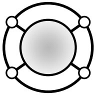
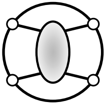
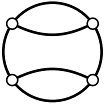
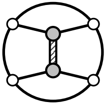
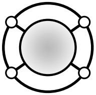
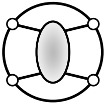
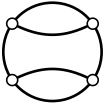
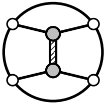
$$\frac{-C_\star}{\Delta-\Delta_\star}\,\delta_{J,J_\star} \in K_{\widetilde{\Delta},J}\,\mathsf{Spec}_s\left[\begin{array}{c}\Delta\\ J\end{array}\right|$$  $$\Big]$$

 $$=$$  $$+\frac{1}{N}$$  $$+\dots$$

$$\mathsf{ope}^2[\mathcal{O}\mathcal{O}\mathcal{O}_\star] \equiv C_\star\left(\frac{1}{N}\right) = C_\star^{(\mathrm{MFT})} + \frac{1}{N}\,C_\star^{(1)} + O\left(\frac{1}{N^2}\right)$$

Anomalous dimensions as double poles

$$\frac{-C_\star}{\Delta - \Delta_\star}\,\delta_{J,J_\star} \in K_{\widetilde{\Delta},J}\,\mathsf{Spec}_s\left[\begin{array}{c}\Delta \\ J\end{array}\right|$$  $$\Bigg]$$

 $= $  $+\,\frac{1}{N}$  $+\ldots$

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Anomalous dimensions as (double) poles

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Anomalous dimensions as (double) poles

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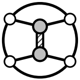
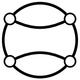
$\gamma^{(1)}_{n,J} = \mathrm{Res}_{\Delta=2\Delta_\phi+2n+J} \Bigg($

Numerator: $\mathsf{Spec}_s\left[\begin{matrix}\Delta \\ J\end{matrix}\right.\Bigg|$  $\Bigg]$

Denominator: $\mathsf{Spec}_s\left[\begin{matrix}\Delta \\ J\end{matrix}\right.\Bigg|$  $\Bigg]$

$\Bigg)$

Crossed channel contributions

Suppose we resolved the "direct" s-channel spectrum.

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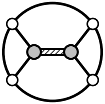
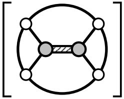


$\longleftrightarrow \quad \mathsf{Spec}_s$



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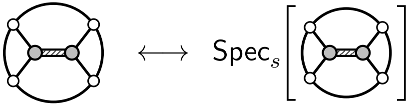

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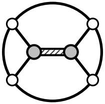
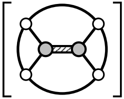
 $\longleftrightarrow$ Spec_s 

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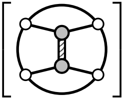
Spec_s 

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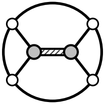

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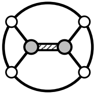

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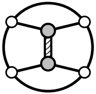

$\Big] = \sum_{\mathcal{O}^{(\mathrm{s})}_{\bullet,0}}$

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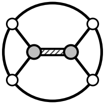
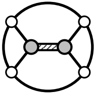
 $\longleftrightarrow$ $\mathsf{Spec}_s\Bigg[$  $\Bigg]$

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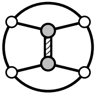
$\mathsf{Spec}_s\Bigg[$  $\Bigg] = \sum_{\mathcal{O}^{(\mathtt{s})}_{\bullet,0}} \mathsf{ope}^2\Big[\mathcal{O}\mathcal{O}\mathcal{O}^{(\mathtt{s})}_{\bullet,0}\Big]$

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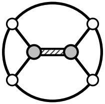
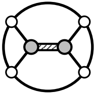
 $\longleftrightarrow$ $\mathsf{Spec}_s\Big[$  $\Big]$

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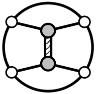
$\mathsf{Spec}_s\Big[$  $\Big] = \sum_{\mathcal{O}^{(\mathtt{s})}_{\bullet,0}} \mathsf{ope}^2\left[\mathcal{O}\mathcal{O}\mathcal{O}^{(\mathtt{s})}_{\bullet,0}\right] \mathsf{CrK}^{\,s \longleftarrow t}_{\left\langle \Delta,J \middle| \Delta^{(\mathtt{s})}_{\bullet,0},0 \right\rangle}$

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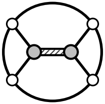
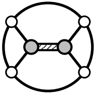
 $\longleftrightarrow$ $\mathsf{Spec}_s\Big[$  $\Big]$

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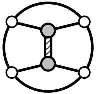
$\mathsf{Spec}_s\Big[$  $\Big]$ $= \sum_{\mathcal{O}^{(\mathtt{S})}_{\bullet,0}} \mathsf{ope}^2\left[\mathcal{O}\mathcal{O}\mathcal{O}^{(\mathtt{S})}_{\bullet,0}\right] \mathsf{CrK}^{\;s \longleftarrow t}_{\left\langle \Delta,J \middle| \Delta^{(\mathtt{S})}_{\bullet,0},0\right\rangle}$

$\mathsf{CrK}^{s\leftarrow t}$ calculated in J. Liu, E. Perlmutter, V. Rosenhaus and D. Simmons-Duffin, *d-dimensional SYK, AdS Loops, and $6j$ Symbols*, *JHEP* **03** (2019) 052 [1808.00612] for $d = 2$ and $d = 4$

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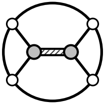
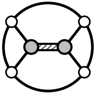
 $\longleftrightarrow$ $\mathsf{Spec}_s\Big[$  $\Big]$

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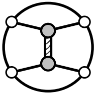
$\mathsf{Spec}_s\Big[$  $\Big]$ $= \sum_{\mathcal{O}^{(\mathtt{S})}_{\bullet,0}} \mathsf{ope}^2\left[\mathcal{O}\mathcal{O}\mathcal{O}^{(\mathtt{S})}_{\bullet,0}\right] \mathsf{CrK}^{\,s\longleftarrow t}_{\langle \Delta,J | \Delta^{(\mathtt{S})}_{\bullet,0},0\rangle}$

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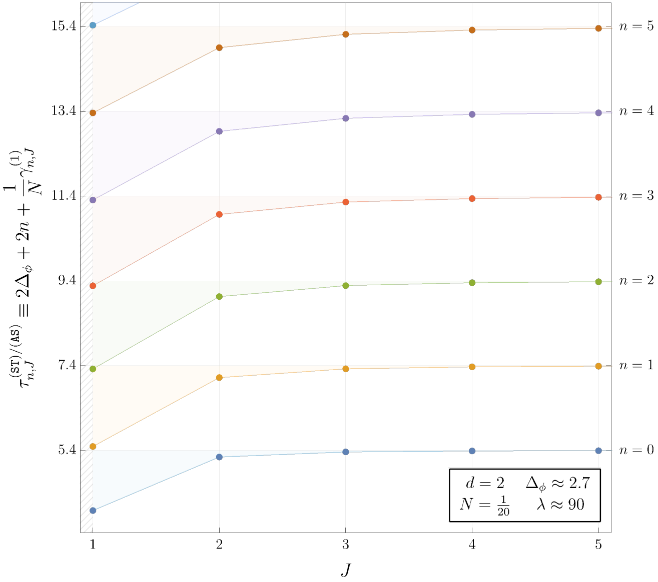
$$\gamma^{(\mathtt{ST})/(\mathtt{AS})}_{n,J} = \sum_{\mathcal{O}^{(\mathtt{S})}_{\bullet,0}} \mathsf{ope}^2\left[\mathcal{O}\mathcal{O}\mathcal{O}^{(\mathtt{S})}_{\bullet,0}\right] \gamma^{(1)}_{n,J}\Big|_{\substack{t\text{-channel} \\ \text{exchange of } \mathcal{O}^{(\mathtt{S})}_{\bullet,0}}}$$

Non-singlet sector — twist–spin plot



Non-singlet sector — large J asymptotics

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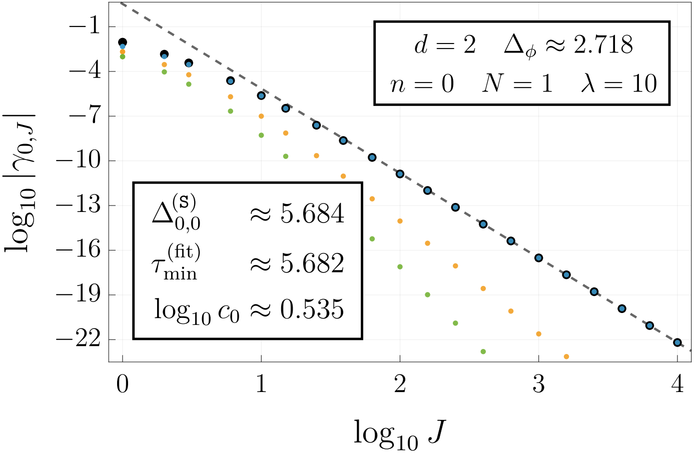

Summary and outlook

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Results:

- Presented general formulas (in $d = 2$ and $d = 4$) for

$$\text{t-chanel conformal block} \xrightarrow{\text{contribution}} \text{anomalous dimensions of s-channel double-twist operators}$$

- Analyzed non-singlet sector of boundary CFT corresponding to the O(N) model in EAdS

  ⇝ complete picture of $\frac{1}{N}$ corrections to the CFT data

Future directions:

- Resolve some technical details — calculation of OPE coefficients, $J = 0$ (ST) operators
- Applications to BCFT or DCFT setting, or other models

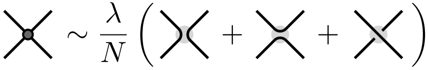
More details in `2503.16345 [hep-th]`

Extra slides

Some extra slides.

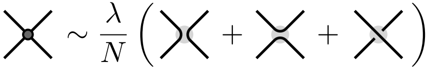
Hubbard–Stratonovich transformation

$$\mathcal{S}[\phi^{\bullet}] = \int_{\mathcal{M}} \mathrm{d}^{d+1}x\,\sqrt{g}\left[\frac{1}{2}(\partial\phi^{\bullet})^2 + \frac{1}{2}m^2(\phi^{\bullet})^2 + \frac{\lambda}{2N}\Big((\phi^{\bullet})^2\Big)^2\right]$$



Hubbard–Stratonovich transformation

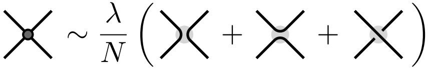
$$\mathcal{S}[\phi^{\bullet}] = \int_{\mathcal{M}} \mathrm{d}^{d+1}x \, \sqrt{g} \left[\frac{1}{2}(\partial\phi^{\bullet})^2 + \frac{1}{2} m^2 (\phi^{\bullet})^2 + \frac{\lambda}{2N} \left((\phi^{\bullet})^2 \right)^2 \right]$$



$\mathcal{S}_{\mathrm{HS}}[\phi^{\bullet}, \sigma]$

Hubbard–Stratonovich transformation

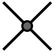
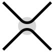
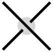
$$\mathcal{S}[\phi^{\bullet}] = \int_{\mathcal{M}} \mathrm{d}^{d+1}x\, \sqrt{g} \left[\frac{1}{2} (\partial \phi^{\bullet})^2 + \frac{1}{2} m^2 (\phi^{\bullet})^2 + \frac{\lambda}{2N} \left((\phi^{\bullet})^2 \right)^2 \right]$$



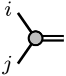
$$\mathcal{S}_{\mathrm{HS}}[\phi^{\bullet}, \sigma] = \int_{\mathcal{M}} \mathrm{d}^{d+1}x\, \sqrt{g} \left[\frac{1}{2} (\partial \phi^{\bullet})^2 + \frac{1}{2} m^2 (\phi^{\bullet})^2 - \frac{1}{2\lambda} \sigma^2 + \frac{1}{\sqrt{N}} \sigma (\phi^{\bullet})^2 \right]$$

Hubbard–Stratonovich transformation

$$\mathcal{S}[\phi^{\bullet}] = \int_{\mathcal{M}} \mathrm{d}^{d+1}x \, \sqrt{g} \left[\frac{1}{2} (\partial \phi^{\bullet})^2 + \frac{1}{2} m^2 (\phi^{\bullet})^2 + \frac{\lambda}{2N} \left((\phi^{\bullet})^2 \right)^2 \right]$$

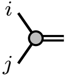
 $\sim \frac{\lambda}{N} \Bigg($  $+$  $+$  $\Bigg)$

$$\mathcal{S}_{\mathrm{HS}}[\phi^{\bullet}, \sigma] = \int_{\mathcal{M}} \mathrm{d}^{d+1}x \, \sqrt{g} \left[\frac{1}{2} (\partial \phi^{\bullet})^2 + \frac{1}{2} m^2 (\phi^{\bullet})^2 - \frac{1}{2\lambda} \sigma^2 + \frac{1}{\sqrt{N}} \sigma (\phi^{\bullet})^2 \right]$$

 $\equiv -\lambda \mathbb{1}$ 

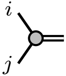
$\equiv \frac{2}{\sqrt{N}} \delta^{ij}$

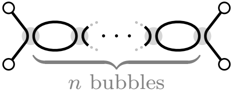
Exact $\boldsymbol{\sigma}$-propagator

 $\equiv -\lambda \mathbb{1}$ 

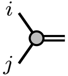
$\equiv \frac{2}{\sqrt{N}}\,\delta^{ij}$

Exact σ-propagator

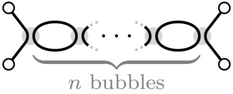
 $\equiv -\lambda \mathbb{1}$  $\equiv \frac{2}{\sqrt{N}}\,\delta^{ij}$

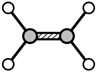
$\sum_{n=0}^{\infty}$ 

Exact $\boldsymbol{\sigma}$-propagator

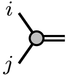
 $\equiv -\lambda \mathbb{1}$ 

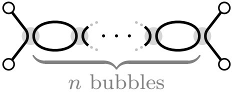
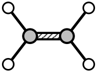
$\equiv \frac{2}{\sqrt{N}}\,\delta^{ij}$

$\sum_{n=0}^{\infty}$ 

$=$  $\sim \frac{1}{N}$

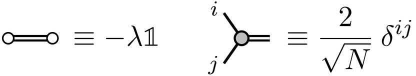
Exact $\boldsymbol{\sigma}$-propagator

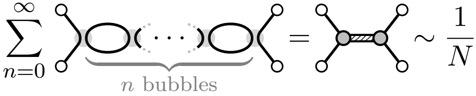
 $\equiv -\lambda\mathbb{1}$  $\equiv \frac{2}{\sqrt{N}}\,\delta^{ij}$

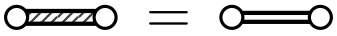
$\sum_{n=0}^{\infty}$  $=$  $\sim \frac{1}{N}$

 $=$

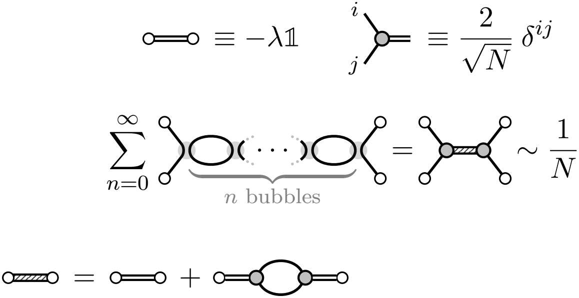
Exact σ-propagator



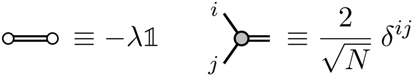


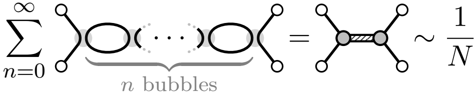


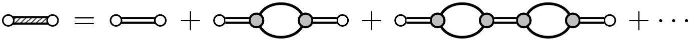
Exact $\boldsymbol{\sigma}$-propagator



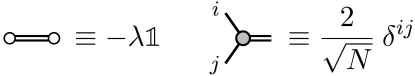
Exact σ-propagator

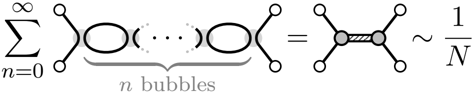


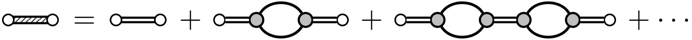




Exact $\boldsymbol{\sigma}$-propagator

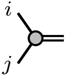




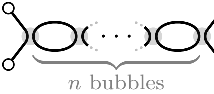


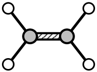
$$= (-\lambda\mathbb{1}) + (-\lambda\mathbb{1}) \circ 2B \circ (-\lambda\mathbb{1}) + \ldots$$

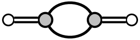
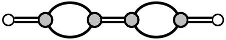
Exact $\boldsymbol{\sigma}$-propagator

 $\equiv -\lambda\mathbb{1}$ 

$\equiv \dfrac{2}{\sqrt{N}}\,\delta^{ij}$

$\sum_{n=0}^{\infty}$ 

$=$  $\sim \dfrac{1}{N}$

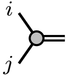
 $=$  $+$  $+$  $+\cdots$

$$= (-\lambda\mathbb{1}) + (-\lambda\mathbb{1}) \circ 2B \circ (-\lambda\mathbb{1}) + \ldots$$

$$= -\lambda \sum_{n=0}^{\infty} (-2\lambda B)^n$$

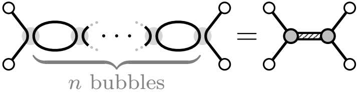
Exact $\boldsymbol{\sigma}$-propagator

$$\equiv -\lambda\mathbb{1}$$

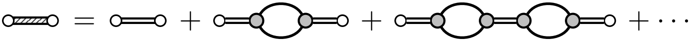


$$\equiv \frac{2}{\sqrt{N}}\,\delta^{ij}$$

$$\sum_{n=0}^{\infty}$$



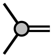
$$\sim \frac{1}{N}$$



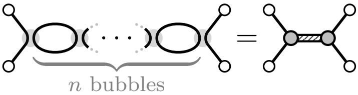
$$= (-\lambda\mathbb{1}) + (-\lambda\mathbb{1}) \circ 2B \circ (-\lambda\mathbb{1}) + \dots$$

$$= -\lambda \sum_{n=0}^{\infty} (-2\lambda B)^n = -\left[\frac{\mathbb{1}}{\lambda} + 2B\right]^{-1}$$

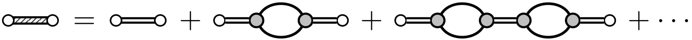
Exact $\boldsymbol{\sigma}$-propagator

 $\equiv -\lambda\mathbb{1}$ 

$\equiv \dfrac{2}{\sqrt{N}}\,\delta^{ij}$

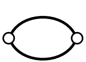
$\sum_{n=0}^{\infty}$ 

$\sim \dfrac{1}{N}$



$$= (-\lambda\mathbb{1}) + (-\lambda\mathbb{1}) \circ 2B \circ (-\lambda\mathbb{1}) + \dots$$

$$= -\lambda \sum_{n=0}^{\infty} (-2\lambda B)^n = -\left[\frac{\mathbb{1}}{\lambda} + 2B\right]^{-1}$$

$B(x,y) \equiv \dfrac{1}{2}\; x$  $y \;\equiv \left[\dfrac{1}{(-\Box + m_\phi^2)\mathbb{1}}\,(x,y)\right]^2$

Utilizing the spectral representation

$$B(x,y) = \int_{\mathbb{R}} \mathrm{d}\nu \, \widetilde{B}(\Delta)\Omega_{\Delta}(x,y) \qquad \left(\Delta \equiv \frac{d}{2} + \mathring{\imath}\nu\right)$$

Utilizing the spectral representation

$$B(x,y) = \int_{\mathbb{R}} \mathrm{d}\nu \, \widetilde{B}(\Delta)\Omega_\Delta(x,y) \qquad \left(\Delta \equiv \frac{d}{2} + i\nu\right)$$

$$= -\left[\frac{\mathbb{1}}{\lambda} + 2B\right]^{-1}$$

Utilizing the spectral representation

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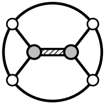
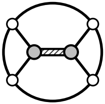


Utilizing the spectral representation

$$B(x,y) = \int_{\mathbb{R}} \mathrm{d}\nu\, \widetilde{B}(\Delta)\Omega_\Delta(x,y) \qquad \left(\Delta \equiv \frac{d}{2} + i\nu\right)$$

$$= -\left[\frac{\mathbb{1}}{\lambda} + 2B\right]^{-1} = \int_{\mathbb{R}} \mathrm{d}\nu \left(\frac{-1}{\lambda^{-1} + 2\widetilde{B}(\Delta)}\right)\Omega_\Delta$$



Utilizing the spectral representation

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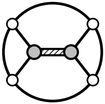
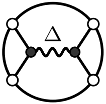


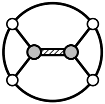
$$= 4 \int_{\mathbb{R}} \mathrm{d}\nu \left(\frac{-1}{\lambda^{-1} + 2\widetilde{B}(\Delta)}\right)$$



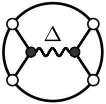
Utilizing the spectral representation

$$B(x,y) = \int_{\mathbb{R}} \mathrm{d}\nu \, \widetilde{B}(\Delta)\Omega_\Delta(x,y) \qquad \left(\Delta \equiv \frac{d}{2} + i\nu\right)$$

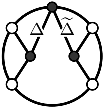
$$= -\left[\frac{\mathbb{1}}{\lambda} + 2B\right]^{-1} = \int_{\mathbb{R}} \mathrm{d}\nu \left(\frac{-1}{\lambda^{-1} + 2\widetilde{B}(\Delta)}\right)\Omega_\Delta$$



$$= 4 \int_{\mathbb{R}} \mathrm{d}\nu \left(\frac{-1}{\lambda^{-1} + 2\widetilde{B}(\Delta)}\right)$$



$$= 4 \int_{\mathbb{R}} \mathrm{d}\nu \left(-\frac{1}{\lambda^{-1} + 2\widetilde{B}(\Delta)}\right)\sqrt{\mathfrak{C}_\Delta \mathfrak{C}_{\widetilde{\Delta}}}\, \frac{\nu^2}{\pi}$$



Utilizing the spectral representation

$$B(x,y) = \int_{\mathbb{R}} \mathrm{d}\nu \, \widetilde{B}(\Delta)\Omega_\Delta(x,y) \qquad \left(\Delta \equiv \frac{d}{2} + \mathring{\imath}\nu\right)$$

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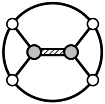
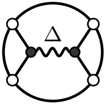
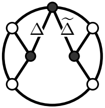
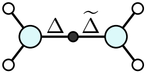
 $= 4\int_{\mathbb{R}} \mathrm{d}\nu \left(\frac{-1}{\lambda^{-1} + 2\widetilde{B}(\Delta)}\right)$ 

$= 4\int_{\mathbb{R}} \mathrm{d}\nu \left(-\frac{1}{\lambda^{-1} + 2\widetilde{B}(\Delta)}\right)\sqrt{\mathfrak{C}_\Delta \mathfrak{C}_{\widetilde{\Delta}}}\,\frac{\nu^2}{\pi}$ 

$= \int_{\frac{d}{2}+\mathring{\imath}\mathbb{R}_{\geq 0}} \frac{\mathrm{d}\Delta}{2\pi \mathring{\imath}} \left(-\frac{1}{\lambda^{-1} + 2\widetilde{B}(\Delta)}\right)\left(\frac{\Gamma_{\dots}^2 \cdots \Gamma_{\dots}}{\cdots \Gamma_{\dots}^2 \cdots \Gamma_{\dots}}\right)$ 

Utilizing the spectral representation

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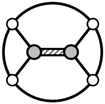
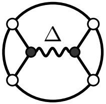
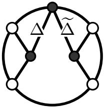


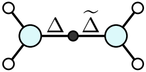
$$= 4\int_{\mathbb{R}} \mathrm{d}\nu \left(\frac{-1}{\lambda^{-1} + 2\widetilde{B}(\Delta)}\right)$$



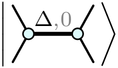
$$= 4\int_{\mathbb{R}} \mathrm{d}\nu \left(-\frac{1}{\lambda^{-1} + 2\widetilde{B}(\Delta)}\right)\sqrt{\mathfrak{C}_\Delta \mathfrak{C}_{\widetilde{\Delta}}}\ \frac{\nu^2}{\pi}$$



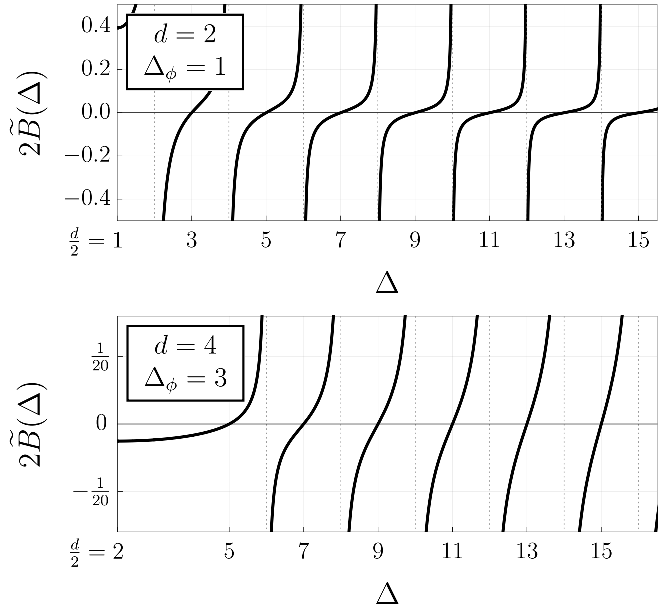
$$= \int_{\frac{d}{2}+i\mathbb{R}_{\geq 0}} \frac{\mathrm{d}\Delta}{2\pi i} \left(-\frac{1}{\lambda^{-1} + 2\widetilde{B}(\Delta)}\right)\left(\frac{\Gamma^2_{\dots}\cdots\Gamma_{\dots}}{\cdots\Gamma^2_{\dots}\cdots\Gamma_{\dots}}\right)$$



$$\equiv \int_{\frac{d}{2}+i\mathbb{R}_{\geq 0}} \frac{\mathrm{d}\Delta}{2\pi i} \left(-\frac{1}{\lambda^{-1} + 2\widetilde{B}(\Delta)}\right)\left(\frac{\Gamma^2_{\dots}\cdots\Gamma_{\dots}}{\cdots\Gamma^2_{\dots}\cdots\Gamma_{\dots}}\right)$$



Criticality in the bulk



Criticality in the bulk